Math 6000, Fall 2020 (Prof. Kinser), Study Checks

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0. Read Chapter 18 - Dummitt and Foote

SE 1. Theorem: There is an equivalence of categories as follows: $\text{Rep}(G) \equiv FG - \mod$.
Prove this is as much detail to master the definitions. See pg. 842-843.

SE 2. (not totally trivial) $\mathbb{Q}_8$ has no faithful two-dimensional **real** representations.

SE.3 Corollary 2 (to Maschke's Theorem) In the setup of the theorem, every finitely generated FG-module is completely reducible.

Prove: Study exercise using inducting on $\dim V$.

SE 4. (a) Prove that if $S \to T$ is a ring homomorphism $\&_T N$ is simple, then ${}_S N$ (by restriction of scalars) also simple.

(b)Show that if $S = S_1 \times S_2$ is a product of rings, $\&_S N$ is simple, then either S_1 or S_2 acts by 0 or ${}_S N$ so ${}_S N$ can be regarded as a simple left S_1 or S_2-module.

SE 5. (In proof of Wedderburn (pg. 4), Sine $Mv \subset Rv \subset E$, get $Rv = Mv \oplus (M' \cap Rv)$.

SE 6. The map $R \to \mathrm{End}_R(E)$ defined by $r \mapsto \lambda r$ is a ring homomorphism.

SE 7. For any $v, w \in V$ as above, thought of $n \times 1$ column vectors using basis above, we have $B(v, w) = v^T \Gamma w \in F$, where v^T is $1 \times n$, Γ is $n \times n$, w is $n \times 1$ and F is 1×1.

SE 8. B is nondegenerate $\iff$ associated matrix Γ as above $\det \Gamma \neq 0$.

SE 9. If B is non-degenerate skew-symmetric form on a vector space V , then $\dim V$ is even.

SE 10. (Hard exercise) Let B be a symmetric nondegenerate bilinear form on V over F.

(1) If $F = \mathbb{R}$, then $O(V, B) \equiv O(p, q)$ for some p, q uniquely determined by B (i.e. can choose basis of V such that B represented by $I_{p,q}$).

(2) If $F = \mathbb{C}$, then $O(V, B) \equiv O(n, \mathbb{C})$ i.e. can choose basis of V such that B represented by Id matrix.